TABLE OF CONTENTS

Preface 16
Introduction 17
Section 1: Value, Order, and Factors 19
1.1 Numbers and the Number Line 20
1.1.1 The Number Line: Understanding Real Numbers 20
What Are Real Numbers? 20
Key Concepts 20
How Does the Number Line Work? 20
1.1.2 ★ Interval Notation 22
★ Exclusive vs. Inclusive Intervals 22
★ Half-Open Intervals 22
★ Unbounded Intervals 22
1.1.3 Absolute Value 22
What Is Absolute Value? 22
★ Distance Between Two Numbers 23
1.1.4 Triangle Inequality 24
Triangle Inequality Theorem 24
1.1.5 ★ Absolute Value Inequality Rules 24
★ "Less Than" Inequality ($|x - a| < b$) 24
★ "Greater Than" Inequality ($|x - a| > b$) 25
1.1.6 Summary of Key Rules and Formulas 26
1) Absolute Value Definition 26
2) ★ Distance Between Two Numbers 26
3) Triangle Inequality Theorem 26
4) ★ Absolute Value Inequality Rules 26
"Less Than" Inequality ($|x - a| < b$) 26
"Greater Than" Inequality ($|x - a| > b$) 26
1.1.7 ★ GMAT-Style Questions 27
1.1.8 ★ Answers 29
1.2 Factors, Multiples, Divisibility, and Remainders 32
1.2.1 Integers and Consecutive Integers 32
★ Properties of Consecutive Integers 32
1.2.2 Factors and Multiples 33
Factors 33
Multiples 33
Prime Numbers 33
GCD (Greatest Common Divisor) 33
LCM (Least Common Multiple) 33
★ Properties of Prime Numbers 34
1.2.3 Division, Quotient, and Remainder 35
★ Modular Exponentiation Rule 35

1.2.4 Even and Odd Integers 37
1.2.5 Prime and Composite Numbers 38
1.2.6 Summary of Key Rules and Formulas 40
1) Integers and Consecutive Integers 40
Definition of Integers 40
Consecutive Integers 40
★ Sum of Consecutive Integers 40
★ Product of Consecutive Integers 40
★ Divisibility by 3 in Consecutive Integers 40
2) Factors and Multiples 40
Definition of Factors 40
Definition of Multiples 40
GCD (Greatest Common Divisor) 40
LCM (Least Common Multiple) 40
★ Primes Property 41
3) Division Equation 41
4) ★ Modular Exponentiation Rule 41
5) Even and Odd Integers 41
Even Numbers 41
Odd Numbers 41
Key Properties of Even and Odd Numbers 41
6) Prime and Composite Numbers 42
Prime Numbers 42
Composite Numbers 42
Prime Factorization 42
1.2.7 ★ GMAT-Style Questions 43
1.2.8 ★ Answers 45
1.3 Exponents 48
1.3.1 Exponents 48
1.3.2 Negative Exponents 48
1.3.3 Fractional (Rational) Exponents 49
How It Works 49
1.3.4 Square Roots and Cube Roots 50
Square Roots 50
Cube Roots 50
1.3.5 Basic Exponent Rules 51
1.3.6 ★ Exponential Growth and Decay 52
1.3.7 Imaginary Numbers 53
Definition of the Imaginary Unit i 53
★ Key Properties of i 53
1.3.8 Summary of Key Rules and Formulas 55
1) Exponentiation Definition 55
2) Special Exponents 55
Squaring a Number 55

Cubing a Number 55
Negative Exponent Rule 55
Fractional Exponent Rule 55
3) Square and Cube Roots 55
The Definition of a Square Root 55
The Definition of a Cube Root 55
Absolute Value Relationship 56
4) Basic Exponent Rules 56
5) ★ Exponential Growth and Decay 56
6) Imaginary Numbers 56
Definition of the Imaginary Unit i 56
★ Key Properties of i 56
1.3.9 ★ GMAT-Style Questions 58
1.3.10 ★ Answers 61
1.4 Decimals and Place Value 64
1.4.1 Decimal Representation 64
1.4.1.1 Understanding Decimals 64
Definition 64
How it works 64
1.4.1.2 Place Values in Decimals 64
1.4.2 Scientific Notation 65
Definition 65
Conversion Rules 65
1.4.3 Basic Operations with Decimals 66
1.4.3.1 Adding and Subtracting Decimals 66
1.4.3.2 Multiplying Decimals 69
1.4.3.3 Dividing Decimals 70
1.4.4 Summary of Key Rules and Formulas 72
1) Scientific Notation 72
2) Conversion Rule 72
3) Steps for Adding and Subtracting Decimals 72
4) Steps for Multiplying Decimals 72
5) Steps for Dividing Decimals 73
1.4.5 ★ GMAT-Style Questions 74
1.4.6 ★ Answers 75
1.5 Properties of Operation 77
1.5.1 Commutative Property 77
Addition 77
Multiplication 77
1.5.2 Associative Property 77
Addition 78
Multiplication 78
1.5.3 Distributive Property 78
Over Addition 78

Over Subtraction 79
1.5.4 Identity Property 79
Addition Identity 80
Multiplication Identity 80
1.5.5 Zero Property of Multiplication 80
Multiplication 80
1.5.6 Inverse Property 80
Additive Inverse 80
Multiplicative Inverse 81
1.5.7 Sign Rules for Multiplication 81
1.5.8 Other Properties 82
Symmetric Property 82
Transitive Property 82
Reflexive Property 82
Substitution Property 82
1.5.9 Exponentiation 83
1.5.10 Summary of Key Rules and Formulas 84
Properties of Arithmetic Operations 84
1) Commutative Property 84
2) Associative Property 84
3) Distributive Property 84
4) Identity Property 84
5) Zero Property of Multiplication 84
6) Inverse Property 84
7) Symmetric Property 84
8) Transitive Property 84
9) Reflexive Property 84
10) Substitution Property 85
1.5.11 ★ GMAT-Style Questions 86
1.5.12 ★ Answers 88
Section 2: Algebra, Equalities, and Inequalities 90
2.1 Algebraic Expressions and Equations 91
2.1.1 Algebraic Expressions 91
2.1.2 Translating Words into Algebraic Expressions 92
2.1.3 Terms and Polynomials 92
2.1.4 Simplifying Expressions 93
2.1.5 Fractions in Algebra 93
2.1.6 Multiplying Algebraic Expressions 94
2.1.7 Evaluating Expressions 94
2.1.8 Algebraic Equations 94
2.1.9 GMAT Shortcuts 95
2.1.10 Summary of Key Rules and Formulas 98
1) Algebraic Expression 98
2) Polynomials 98

3) Fractions in Algebra 98
4) Multiplying Algebraic Expressions 99
5) Linear Equation Formula 99
6) Quadratic Equation Formula 99
7) Perfect Square Formulas 99
8) Difference of Squares 99
9) ★ Sum/Difference of Cubes 99
2.1.11 ★ GMAT-Style Questions 100
2.1.12 ★ Answers 102
2.2 Linear Equations 104
2.2.1 Standard Form of a Linear Equation 104
2.2.2 Linear Equations with One Unknown 104
2.2.3 Solving Linear Equations with One Unknown 105
2.2.4 Linear Equations with Two Unknowns 106
2.2.5 Methods to Solve Two Linear Equations with Two Unknowns 107
Substitution Method 107
Elimination Method 108
★ Graphical Method 108
2.2.6 Slope-Intercept Form of a Linear Equation 110
2.2.7 ★ Point-Slope Form of a Linear Equation 111
2.2.8 Slope of a Line 113
2.2.9 Intercepts 114
2.2.10 ★ Applications of Linear Equations 114
2.2.11 Summary of Key Rules and Formulas 116
1) Linear Equation with One Unknown 116
2) Solving a Linear Equation with One Unknown 116
3) Linear Equation with Two Unknowns 116
4) Slope-Intercept Form of a Linear Equation 116
5) ★ Point-Slope Form of a Linear Equation 116
6) Slope Formula of a Linear Equation 117
7) X-Intercept of a Linear Equation 117
8) Y-Intercept of a Linear Equation 117
9) Solving a System of Linear Equations 117
2.2.12 ★ GMAT-Style Questions 118
2.2.13 ★ Answers 120
2.3 Factoring and Quadratic Equations 122
2.3.1 What is Factoring? 122
2.3.2 Factoring to Solve Equations 122
Solving Polynomial Equations by Factoring 122
Solving Rational Equations by Factoring 123
2.3.3 Types of Quadratic Factoring 123
★ Trinomial (AC Method) 124
Difference of Squares 124
Perfect Square Trinomial 125

2.3.4 Solving Quadratic Equations by Factoring 125
2.3.5 ★ Completing the Square 126
2.3.6 Quadratic Formula 127
2.3.7 ★ How to Recognize Which Factoring Method to Use 128
2.3.8 Summary of Key Rules and Formulas 130
1) Solving Rational Equations by Factoring 130
2) Standard Form of a Quadratic Equation 130
3) ★ Completing the Square Method 130
Common Factoring Patterns for Quadratic Equations 130
4) ★ Factoring Trinomials (AC Method) 130
5) Quadratic Formula 130
6) Perfect Square Trinomial 131
7) Difference of Squares 131
2.3.9 ★ GMAT-Style Questions 132
2.3.10 ★ Answers 134
2.4 Inequalities 137
2.4.1 Introduction to Inequalities 137
2.4.2 Types of Inequalities 137
Linear Inequality 137
Quadratic Inequality 137
Compound Inequality 137
2.4.3 Solving Linear Inequalities 138
2.4.5 ★ Solving Quadratic Inequalities 139
2.4.6 ★ Absolute Value Inequalities 140
2.4.7 Summary of Key Rules and Formulas 142
1) Inequality Types 142
2) Solving Linear Inequalities 142
3) ★ Solving Quadratic Inequalities 143
4) ★ Solving Absolute Value Inequalities 143
2.4.8 ★ GMAT-Style Questions 145
2.4.9 ★ Answers 147
2.5 Functions 150
2.5.1 What is a Function? 150
2.5.2 Types of Functions 150
Linear Function 150
Quadratic Function 151
★ Polynomial Function 151
★ Rational Function 151
★ Exponential Function 152
★ Logarithmic Function 152
2.5.3 How to Evaluate a Function 153
2.5.4 ★ Function Operations 154
Addition 154
Subtraction 154

Multiplication 154
Division 154
2.5.5 ★ Inverse of a Function 155
2.5.6 ★ Understanding Graphs of Functions 155
Intercepts 155
Symmetry 156
Asymptotes 158
2.5.7 ★ Piecewise Function 159
2.5.8 Summary of Key Rules and Formulas 160
1) General Form of a Function 160
2) General Form of a Linear Function 160
3) General Form of a Quadratic Function 160
4) ★ General Form of a Polynomial Function 160
5) ★ General Form of a Rational Function 160
6) ★ General Form of an Exponential Function 160
7) ★ General Form of a Logarithmic Function 161
8) ★ Inverse Function 161
9) ★ Function Operations 161
10) ★ Piecewise Function 161
2.5.9 ★ GMAT-Style Questions 162
2.5.10 ★ Answers 164
2.6 Graphing 166
2.6.1 The Basics of Graphing 166
The Coordinate Plane 166
2.6.2 Plotting Points 167
2.6.3 ★ Distance Between Two Points 167
Distance Formula 167
2.6.4 Linear Equations and Graphing Lines 168
Graphing Linear Equations 168
Intercepts 173
Linear Intersections 174
Finding the Slope between two points 177
2.6.5 ★ Quadratic Functions and Parabolas 178
Key Features of the Parabola in a Quadratic Function 180
2.6.6 Graphing Functions 182
How to Graph a Function 182
2.6.7 Summary of Key Rules and Formulas 184
1) ★ Distance Formula 184
2) Equation of a Line (Slope-Intercept Form) 184
3) Slope Calculation 184
4) X-Intercept of a Linear Equation 185
5) Y-Intercept of a Linear Equation 185
6) ★ the Vertex of a Quadratic Function 185
7) ★ The Axis of Symmetry of a Quadratic Function 186

8) ★ Intercepts of a Quadratic Function 186
2.6.8 ★ GMAT-Style Questions 187
2.6.9 ★ Answers 189
2.7 Formulas and Measurement Conversion 192
2.7.1 What is a Formula? 192
2.7.2 Measurement Conversion as a Formula 192
2.7.3 Common Formulas for the GMAT 193
2.7.3.1 Arithmetic 193
2.7.3.2 Ratios and Proportions 194
2.7.3.3 Rates and Work 194
2.7.4 Unit Conversions 195
2.7.5 Summary of Key Rules and Formulas 196
1) Unit Conversions 196
Common Formulas for the GMAT 196
2) Arithmetic 196
3) Ratios and Proportions 196
4) Rates and Work 196
2.7.6 ★ GMAT-Style Questions 198
2.7.7 ★ Answers 200
Section 3: Rates, Ratios, and Percents 202
3.1 Ratio and Proportion 203
3.1.1 What is a Ratio? 203
Notation 203
★ Key Properties of Ratios 203
3.1.2 What is a Proportion? 204
Notation 204
Cross Multiplication Property of Proportions 204
3.1.3 ★ Applications of Ratios and Proportions 204
3.1.4 ★ Direct and Inverse Proportions 206
★ Direct Proportion 206
★ Inverse Proportion 207
3.1.5 The Unitary Method 208
3.1.6 Summary of Key Rules and Formulas 209
1) Ratio Definition 209
2) ★ Simplifying Ratios 209
3) ★ Part-to-Whole and Whole-to-Part Ratio 209
4) ★ Dividing a Quantity in a Ratio 209
5) Proportion Definition 209
6) Cross Multiplication in Proportions 209
7) Finding an Unknown in a Proportion 210
8) ★ Direct and Inverse Proportion 210
3.1.7 ★ GMAT-Style Questions 211
3.1.8 ★ Answers 214
3.2 Fractions 218

3.2.1 Understanding Fractions 218
Types of Fractions 218
3.2.2 Simplifying Fractions 219
3.2.3 Converting Between Mixed Numbers and Improper Fractions 219
Mixed to Improper Fraction 219
Improper to Mixed 220
3.2.4 Adding and Subtracting Fractions 221
Same Denominator 221
Different Denominators 221
3.2.5 Multiplying Fractions 222
3.2.6 Dividing Fractions 223
3.2.7 ★ Comparing Fractions 223
3.2.8 Summary of Key Rules and Formulas 225
1) Definition of Fraction 225
2) ★ Converting Mixed Numbers to Improper Fractions 225
3) ★ Converting Improper Fractions to Mixed Numbers 225
4) Simplifying Fractions 225
5) Addition and Subtraction 226
6) Multiplication 226
7) Division 226
8) ★ Comparing Fractions 226
3.2.9 ★ GMAT-Style Questions 227
3.2.10 ★ Answers 230
3.3 Percents 234
3.3.1 What is a Percent? 234
3.3.2 Key Percent Formulas 234
Percent Formula 234
Finding the Part 234
★ Finding the Whole 235
3.3.3 Types of Percent Problems 236
Basic Percent Problems 236
Percent Increase and Decrease 236
Percent Change Between Two Numbers 237
Discounts 238
★ Markups 238
3.3.4 Summary of Key Rules and Formulas 240
1) General Percent Formula 240
2) Percentage of a Number 240
3) ★ Finding the Whole (Total Amount) 240
4) Percent Increase Formula 240
5) Percent Decrease Formula 240
6) Percent Change Formula 240
7) Percent of a Percent 241
8) Discount Formula (for calculating price after discount) 241

9) ★Markup Formula (for calculating price after markup) 241
3.3.5 ★GMAT-Style Questions 242
3.3.6 ★Answers 243
3.4 Converting Decimals, Fractions, and Percents 245
3.4.1 ★Converting Decimals to Fractions 245
3.4.2 ★Converting Fractions to Decimals 246
3.4.3 ★Converting Decimals to Percents 246
3.4.4 ★Converting Percents to Decimals 247
3.4.5 ★Converting Fractions to Percents 247
3.4.6 Converting Percents to Fractions 248
3.4.7 Decimal as a Sum of Fractions 248
3.4.8 Summary of Key Rules and Formulas 250
1) ★Decimal to Fraction Conversion 250
2) ★Decimal to Percent Conversion 250
3) ★Fraction to Decimal Conversion 250
4) ★Fraction to Percent Conversion 250
5) ★Percent to Decimal Conversion 250
6) Percent to Fraction Conversion 250
7) Decimals as Sums of Fractions 251
3.4.9 ★GMAT-Style Questions 252
3.4.10 ★Answers 254
3.5 Working with Decimals, Fractions, and Percents 257
3.5.1 Gross Profit 257
3.5.2 Simple Annual Interest 258
3.5.3 Compound Interest 258
3.5.4 Using Tables to Solve Percentage and Fraction Word Problems 260
3.5.5 Summary of Key Rules and Formulas 262
1) Gross Profit 262
2) Gross Profit Percentage (based on selling price) 262
3) Simple Interest (Annual) 262
4) Simple Amount (Annual) 262
5) Compound Amount (Annual) 262
6) Compound Interest (Annual) 263
3.5.6 ★GMAT-Style Questions 264
3.5.7 ★Answers 266
3.6 Rate, Work, and Mixture Problems 268
3.6.1 Rate Problems 268
Understanding Rate 268
Key Formulas 268
Types of Rate Problems 269
3.6.2 Work Problems 271
Understanding Work Rates 271
Key Formulas 271
Types of Work Problems 272

3.6.3 Mixture Problems 273
Understanding Mixtures 273
Key Formulas 273
Types of Mixture Problems 274
3.6.4 Summary of Key Rules and Formulas 276
Distance, Rate, and Time 276
1) Distance 276
2) Rate (or speed) 276
3) Time 276
4) ★ Relative Speed for Objects Moving in Opposite Directions 276
5) ★ Relative Speed for Objects Moving in the Same Direction 276
Work Problems 276
6) Work 276
7) Time to Complete Work 276
8) Rate 277
9) Combined Work (working together) 277
Mixture Problems 277
10) Total Value 277
11) Final Concentration (or weighted average) 277
3.6.5 ★ GMAT-Style Questions 278
3.6.6 ★ Answers 280
Section 4: Statistics, Sets, Counting, Probability, Estimation and Series 283
4.1 Statistics 284
4.1.1 Average (Mean) 284
4.1.2 Median 285
How It Works 285
4.1.3 Mode 285
4.1.4 Range 286
4.1.5 Standard Deviation 287
What Does It Mean? 287
How Is Standard Deviation Calculated? 287
★ Key Formulas 287
4.1.6 Frequency and Frequency Distribution 289
What Is Frequency? 290
What Is Frequency Distribution? 290
Frequency Distribution Table 290
★ Grouped Frequency Distribution 290
4.1.7 Summary of Key Rules and Formulas 292
1) Average (Mean) 292
2) Median 292
3) Mode 292
4) Range 292
5) ★ Variance 292
6) ★ Standard Deviation 292

7) Frequency 293
4.1.8 ★ GMAT-Style Questions 294
4.1.9 ★ Answers 296
4.2 Sets 298
4.2.1 Introduction to Sets 298
4.2.2 Number of Elements in a Set 298
4.2.3 Universal Set (U) 299
4.2.4 Subset 299
4.2.5 Set Operations 300
Intersection ($A \cap B$) 300
Union ($A \cup B$) 300
★ Difference ($A - B$) 300
★ Complement (A') 301
4.2.6 Counting Elements in Unions 301
Union of Two Sets 301
★ Union of Three Sets 302
★ General Case: Union of n Sets 302
4.2.7 Venn Diagrams 302
Common Regions in a Venn Diagram 303
4.2.8 Summary of Key Rules and Formulas 306
1) Set Notation 306
2) Cardinality ($|A|$) 306
3) Universal Set (U) 306
4) Subset 306
5) Set Operations 306
6) Counting Elements in Unions 307
7) ★ Counting elements using Venn Diagrams 307
4.2.9 ★ GMAT-Style Questions 309
4.2.10 ★ Answers 311
4.3 Counting Methods 313
4.3.1 What Are Counting Methods? 313
4.3.2 Multiplication Principle: The Foundation of Counting 313
4.3.3 Factorials: The Cornerstone of Counting 314
4.3.4 Combinations: When Order Doesn't Matter 314
4.3.5 Permutations: When Order Matters 315
4.3.6 ★ Common Counting Problem Types and Solving Strategies 316
4.3.6.1 ★ Repeated Element Arrangement Problems 316
4.3.6.2 ★ Selection Problems with "At Least" or "At Most" Conditions 317
4.3.6.3 ★ Grouping with Restrictions 318
4.3.6.4 ★ Circular Arrangement Problems 319
4.3.6.5 ★ Distributing Items into Groups 320
Distinct Groups 320
Identical Groups 321
4.3.7 Summary of Key Rules and Formulas 322

1) Multiplication Principle 322
2) Factorials 322
3) Combinations 322
4) Permutations 323
5) ★ Repeated Element Arrangement 323
6) ★ "At Least" / "At Most" Problems 323
7) ★ Grouping with Restrictions 323
8) ★ Circular Arrangements 323
9) ★ Distributing Items into Groups 324
4.3.8 ★ GMAT-Style Questions 325
4.3.9 ★ Answers 327
4.4 Probability 330
4.4.1 What is Probability? 330
4.4.2 Types of Probability Events 331
4.4.2.1 Independent Events 331
4.4.2.2 Dependent Events 332
4.4.2.3 Mutually Exclusive Events 333
4.4.2.4 Non-Mutually Exclusive Events 334
4.4.3 Probability Rules and Theorems 335
4.4.3.1 The Addition Rule 335
4.4.3.2 The Multiplication Rule 336
4.4.3.3 Complement Rule 336
4.4.4 Conditional Probability 337
4.4.4.1 ★ Law of Total Probability 337
4.4.5 ★ Expected Value 338
4.4.6 ★ Common Question types in GMAT 339
4.4.6.1 ★ Probability of "At Least One" Occurrence 339
4.4.6.2 ★ Probability of "Exactly One" Occurrence 340
4.4.7 Summary of Key Rules and Formulas 343
1) Probability Formula 343
2) Independent Events 343
3) Dependent Events 343
4) Mutually Exclusive Events 343
5) Non-Mutually Exclusive Events 344
6) Addition Rule 344
7) Multiplication Rule 344
8) Complement Rule 344
9) Conditional Probability 345
10) ★ Law of Total Probability 345
11) ★ Expected Value 345
12) ★ Probability of "At Least One" Occurrence 345
13) ★ Probability of "Exactly One" Occurrence 346
4.4.8 ★ GMAT-Style Questions 347
4.4.9 ★ Answers 350

4.5 Estimation 1
4.5.1 Rounding to Place Values 1
How to Round a Number to the Nearest Place Value 1
4.5.2 Rounding to Compatible Numbers 1
How to Use Compatible Numbers 1
4.5.3 Rounding to Powers of Numbers 1
How to Use Powers of Numbers 1
4.5.4 Range Estimation 1
How to Use Range Estimation 1
4.5.5 Summary of Key Rules and Formulas 1
1) Rounding to Place Values 1
2) Rounding to Compatible Numbers 1
3) Rounding to Powers of Numbers 1
4) Range Estimation 1
4.5.6 ★ GMAT-Style Questions 1
4.5.7 ★ Answers 1
4.6 Sequences and Series 1
4.6.1 What Are Sequences and Series? 1
4.6.1.1 Sequence 1
4.6.1.2 Series 1
4.6.2 Key Sequence Types 1
4.6.2.1 Arithmetic Sequence 1
Finding the Sum of an Arithmetic Sequence 1
4.6.2.2 ★ Geometric Sequence 1
★ Finding the Sum of a Geometric Sequence 1
4.6.3 ★ Special Series 1
4.6.3.1 ★ Sum of Consecutive Integers 1
4.6.3.2 ★ Sum of Squares of Integers 1
4.6.3.3 ★ Sum of Cubes of Integers 1
4.6.4 ★ Recognizing Patterns in Non-Standard Sequences and Series 1
4.6.4.1 ★ Recognizing and Working with Non-Standard Sequences 1
4.6.4.2 ★ Smart Techniques for Series (Summations) 1
4.6.5 Summary of Key Rules and Formulas 1
1) General Term of an Arithmetic Sequence 1
2) Sum of an Arithmetic Series (Sequence) 1
3) ★ General Term of a Geometric Sequence 1
4) ★ Sum of a Geometric Series (Sequence) 1
5) ★ Sum of Consecutive Integers 1
6) ★ Sum of Squares of Integers 1
7) ★ Sum of Cubes of Integers 1
8) ★ Odd-Term Arithmetic Series Sum Formula 1
9) ★ Finite Series Approximation Formula 1
4.6.6 ★ GMAT-Style Questions 1
4.6.7 ★ Answers 1